Photoinduced resonant tunneling treated by an extended transfer Hamiltonian method

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A method, originally due to Heitler, is utilized to extend the transfer Hamiltonian description to resonant tunneling for the purpose of calculating transition probabilities and general frequency response characteristics of coupled systems. The scanning tunneling microscope (STM) is treated as an example of a single barrier and an irradiated quantum well as an example of a double barrier. The saturation of the contact resistance in the STM is easily derived and a simple physical explanation for the high-frequency response of an irradiated double junction is presented. In the latter case, it is found that the cutoff in the frequency response for high frequencies is limited by the optical properties of the outer electrodes of the double barrier.

I. INTRODUCTION

One of the most useful schemes for calculating tunneling probabilities is based on the so-called transfer Hamiltonian formalism.[1] Recently, the formalism has been discussed with a view to extending it to situations where there are real intermediate states, such as in double barriers.[2,3] This problem is of current interest in resonant tunneling through double-barrier structures, with or without applied electromagnetic fields, and in the treatment of the scanning tunneling microscope (STM).[4] In the present paper we will give a description of these processes within an extended transfer Hamiltonian formalism. The usual transfer Hamiltonian method yields a divergent current for the case of resonant tunneling and fails for the case of very small tip to sample distance in the STM. This formalism is based on the scheme originally developed by Heitler[5] many years ago. The formalism is illustrated with two systems of interest in device applications: the single barrier and the double barrier shown schematically in Fig. 1. Basically the single barrier has left states (L) and right states (R), and the double barrier also has an intermediate state (I). A crucial feature of the double barrier is a broadening of the intermediate state due to its coupling with the continuum at either side. This is of importance in the context of resonant tunneling quantum wells which exhibit promising characteristics for use in electronics as oscillators or switching devices[6] such as a negative-differential-resistance (NDR) current-voltage relationship. The speed of the device is of considerable importance, i.e., its intrinsic time scales which are intimately connected to the intermediate-state broadening.

A large part of this paper is devoted to a discussion of a quantum well subjected to radiation. In a series of papers Sollner and co-workers[7-11] have used a resonant tunneling diode to perform detection and mixing at frequencies as high as 2.5 THz ($\sim$60 fs), fundamental oscillations at room temperature up to 200 GHz,[8-10] and picosecond switching times.[11] The NDR of the double barrier has also found its use in transistor applications.[12,13] This is remarkable progress in a field which started some twenty years ago with the pioneering superlattice work by Tsu and Esaki,[14] quickly utilizing the advantage of the NDR properties exhibited by a double-barrier quantum-well structure.[15] Naturally this has also stimulated broad theoretical activity in order to understand the fundamental time scales of a resonant tunneling device,[2,9,10,16-35] apart from a general surge of interest in the concept of "time of tunneling."[36]

In 1963 Tien and Gordon[37] treated the case of tunneling under the influence of a photon field. They assumed that the potential due to the photons could be represent-

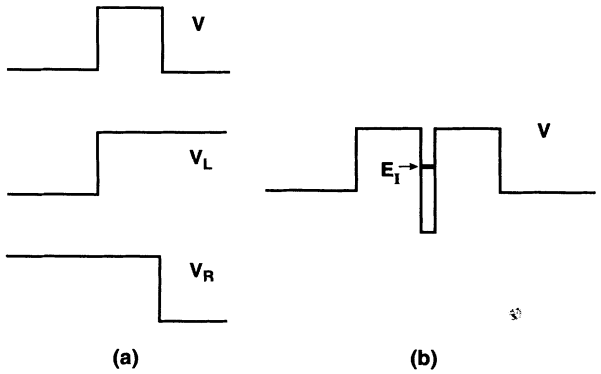

FIG. 1. (a) The full potential V represents the case of a single tunneling barrier. The potential V_L of the left Hamiltonian is indicated. Eigenfunctions of H_L are confined to the left region of space and are used to describe the electron prior to tunneling. The potential V_R of the right Hamiltonian is also shown. Eigenfunctions of H_R describe the electron after tunneling. (b) The potential that describes the double barrier is shown schematically. The resonant level has an energy E_I as denoted in the figure.

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ed by $V\cos(\omega t)$ where V is a spatially constant potential on one side of the junction and is zero on the other side. This simplified Hamiltonian can then be solved exactly and leads to the picture that a state of given energy develops sidebands at multiples of the photon energy. Tunneling takes place from a state of energy E on one side of the junction to a state $E\pm n\hbar\omega$ on the other. This picture is particularly useful for intense photon fields because it allows an exact solution of multiphoton absorption or emission. We are interested in the case of relatively weak photon fields and take the point of view that photoassisted tunneling is a two-step process in which photoabsorption (or emission) can take place either prior to or subsequent to tunneling and the photon field is represented by a potential of the form $W(\mathbf{x},\mathbf{p})\cos(\omega t)$ in the electrodes. Thus the optical response of the electrodes determines the maximum frequency at which the junction can respond to the photons. Previous workers[35] have assumed such a potential to exist only in the barrier region, but the electromagnetic penetration depth is much larger than the barrier region and we assume the potential affects the electrodes rather than the barrier. Finally, it has been suggested[16] that the width of the resonant level γ determines the maximum response frequency. If $\hbar\omega>\gamma$, the electrons cannot follow the external field because γ provides a measure of the time they spend in the resonant level, $\hbar/\gamma$. This point of view will be discussed later.

In Sec. II we introduce Heitler's scheme, within the transfer Hamiltonian formalism, to calculate the current through single and double barriers and develop the necessary formalism to treat the case of an irradiated quantum well. In Sec. III we discuss the Sollner experiment. The paper is summarized in Sec. IV.

II. THEORY

A. Formalism

The transfer Hamiltonian method has served as an extremely valuable approach to tunneling problems. Rather than solving the Schrödinger equation for the entire tunneling junction it is only required to find solutions for the left- and right-hand sides of the junction separately. Furthermore, results are then expressed in terms of properties of the left- and right-hand sides of the junction such as left- and right-hand side densities of states. In this section we develop a multiple-scattering formalism for the transfer Hamiltonian method. This allows the self-energy to be treated correctly to lowest order in the electron tunneling probability with the result that the probability for resonant tunneling in a heterojunction is finite at resonance. The expression for the tunneling probability is shown to be identical to that calculated by the usual method of wave-function matching in the case of a one-dimensional symmetric junction. It will be shown that this method leads to a minimum tunneling resistance $h/2e^2$ for tunneling in a junction. Finally, the case of an external field corresponding to light incident on the junction is treated in this formalism.

The usual transfer Hamiltonian treatment of tunneling through a barrier is as follows. Let the wave functions φ_L governed by a Hamiltonian H_L and energies E_L describe states localized to the left of a barrier and similarly φ_R, H_R, and E_R describe right-hand states. The total potential V as well as the left and right potentials V_L and V_R are shown in Fig. 1(a). The wave functions $\{\varphi_L\}$ are orthonormal as are $\{\varphi_R\}$; however $\langle\varphi_L|\varphi_R\rangle\neq 0$. The system wave function is

$$\psi=\sum_l a_l(t)\varphi_l e^{-iE_l t/\hbar}+\sum_r b_r(t)\varphi_r e^{-iE_r t/\hbar}\ , \tag{1a}$$

where

$$\langle\psi|\psi\rangle=1\ . \tag{1b}$$

At $t=0$ the electron is (say) in the particular left-hand state φ_0 so that at $t=0$

$$a_0(0)=1\ , \tag{2a}$$

$$a_l(0)=b_r(0)=0\quad (l\neq 0)\ . \tag{2b}$$

Use of Eqs. (1) and (2) in $i\hbar\dot{\psi}=H\psi$, where H is the Hamiltonian that describes the *entire* system, yields to lowest order in the overlap

$$i\hbar\dot{b}_R\simeq\langle\psi_R|(H-E_0)|\varphi_0\rangle e^{-i(E_0-E_R)t/\hbar}\ , \tag{3}$$

where $|b_R|^2$ is the probability the electron has tunneled to the state R at the right-hand side of the barrier. It is well known[1] that

$$\langle\phi_R|(H-E_0)|\varphi_0\rangle=\hbar j_{0R}(z_0)\ , \tag{4a}$$

where

$$j_{0R}(z_0)=\frac{\hbar}{2m}\left[\varphi_0\frac{\partial\varphi_R}{\partial z}-\varphi_R\frac{\partial\varphi_0}{\partial z}\right]_{z=z_0} \tag{4b}$$

and z_0 is a point in the barrier.

The matrix element $\langle\varphi_R|(H-E_0)|\varphi_0\rangle$ is a measure of the overlap of φ_R and φ_0 and is small. It is easily shown that in the case of tunneling through a resonant barrier Eq. (3) is generalized to

$$i\hbar\dot{b}_R=\left[\langle\varphi_R|(H-E_0)|\varphi_0\rangle+\frac{\langle\varphi_R|(H-E_0)|\varphi_I\rangle\langle\varphi_I|(H-E_0)|\varphi_0\rangle}{E_0-E_I+i\varepsilon}\right]\times e^{-i(E_0-E_R)t/\hbar}\ , \tag{5}$$

where φ_I is the wave function describing the resonant state in the barrier and E_I is its energy. Near resonance this expression becomes infinite as $E_0\simeq E_I$. In order to treat this case correctly it is necessary to include a self-energy so that the energy denominator in Eq. (5) is replaced by $E_0-E_I-\Sigma_I$ and is not singular at resonance since $\mathrm{Im}(\Sigma_I)\neq 0$. This can be done in a straightforward manner by applying a method developed by Heitler[5] to the transfer Hamiltonian method.

Let the wave functions describing the left system, resonant state, and right system be denoted by φ_n, where two

states are orthogonal only if they refer to the same system. The potentials for these systems are shown schematically in Fig. 1(b). The wave function describing the electron is written as

$$\psi(t)=\sum_n \varphi_n e^{-iE_n t/\hbar} b_n(t) , \tag{6}$$

where ψ satisfies

$$i\hbar\frac{\partial\psi}{\partial t}=H\psi , \tag{7}$$

and where H is the Hamiltonian for the entire system. Use of Eqs. (6) and (7) yields

$$i\hbar\dot{b}_s=\sum_n \langle s|(H-E_n)|n\rangle b_n e^{i(E_s-E_n)t/\hbar} - i\hbar\sum_{n\neq s}\dot{b}_n\langle s|n\rangle e^{i(E_s-E_n)t/\hbar} . \tag{8}$$

A particle is placed in the state $s=0$ at $t=0$ by adding a term $i\hbar\delta_{s,0}\delta(t)$ to the right-hand side of Eq. (8) with the boundary condition that for all n, $b_n(t)=0$ for $t<0$. Equation (8), including the added term, is solved by the use of

$$b_n(t)=\frac{i}{2\pi}\int_{-\infty}^{\infty} dE\, G_{n,0}(E)e^{i(E_n-E)t/\hbar} \tag{9a}$$

and

$$i\hbar\delta(t)=\frac{i}{2\pi}\int_{-\infty}^{\infty} dE\, e^{i(E_0-E)t/\hbar} \tag{9b}$$

to obtain

$$(E-E_s')G_{s,0}(E)=\sum_{n\neq s} H'_{s,n}(E)G_{n,0}(E)+\delta_{s,0} , \tag{10a}$$

where

$$E_s'=\langle s|H|s\rangle \tag{10b}$$

and

$$H'_{s,n}(E)=\langle s|(H-E)|n\rangle . \tag{10c}$$

Note that E_s' differs from E_s because the basis states do not diagonalize the total Hamiltonian. The difference is only second order in the wave-function overlap.

For $s\neq 0$, define

$$G_{s,0}(E)=U_{s,0}(E)G_{0,0}(E)\xi(E-E_s') , \tag{11a}$$

where

$$\xi(E)=\frac{1}{E+i\varepsilon} . \tag{11b}$$

This choice of sign for the term $i\varepsilon$ will ensure $b_n(t)=0$ for $t<0$. Use of Eq. (11a) in Eq. (10a) gives for $s\neq 0$

$$U_{s,0}(E)=H'_s(E)+\sum_{n\neq s,0} H'_{s,n}(E)U_{n,0}(E)\xi(E-E_n') . \tag{12}$$

For the case $s=0$ Eqs. (10a) and (11a) yield

$$G_{0,0}(E)=1/[E-E_0'-\Sigma(E)] , \tag{13a}$$

where

$$\Sigma(E)=\sum_{n\neq 0} H'_{0,n}(E)U_{n,0}(E)\xi(E-E_n') . \tag{13b}$$

The probability of a transition from the state 0 to the state n per unit time is[5]

$$\omega_{n,0}=\frac{2\pi}{\hbar}|U_{n,0}(E_n')|^2\delta(E_n'-E_0') . \tag{14}$$

The quantity $b_n(t)$ can be obtained from Eq. (9a) through the use of Eqs. (11) and (13).

B. Application to STM

This formalism is first applied to the case of a single tunnel junction where experiment shows a saturation.[38] We calculate the tunneling probability for repeated tunneling between the sides of the junction as described by Eq. (12). Let r_0 be a final state on the right-hand side of the barrier [see Fig. 1(a)] and l_0 the initial state of the electron on the left. Equation (12) gives

$$U_{r_0,l_0}(E)=H'_{r_0,l_0}(E)+\sum_{l\neq l_0} H'_{r_0,l}(E)U_{l,l_0}(E)\xi(E-E_l') , \tag{15a}$$

$$U_{l,l_0}(E)=\sum_r H'_{l,r}(E)U_{r,l_0}(E)\xi(E-E_r') . \tag{15b}$$

Assuming no intracoupling, i.e., $\langle a|H'|b\rangle=0$ if a and b belong to the same side of the junction.

In order to solve for U_{r_0,l_0} assume that

$$H'_{r,l}(E)=\langle H'\rangle , \tag{16}$$

independent of r and l. Then

$$U_{r,l_0}(E)\equiv U^R(E) , \tag{17a}$$

$$U_{l,l_0}(E)\equiv U^L(E) , \tag{17b}$$

independent of r and l. Use of Eqs. (16) and (17) in Eq. (15) yields

$$U^R(E)=\frac{\langle H'\rangle}{1-|\langle H'\rangle|^2 G_L(E)G_R(E)} , \tag{18a}$$

where

$$G_L(E)=\sum_l \xi(E-E_l'),\quad G_R(E)=\sum_r \xi(E-E_r') \tag{18b}$$

are the left-hand side and right-hand side Green's functions, respectively.

The total current is

$$I(V)=2e\sum_{l_0,r_0}\omega_{r_0,l_0} =\frac{4\pi e}{\hbar}\sum_{l_0,r_0}|U^R(E_{r_0}')|^2\delta(E_{l_0}'-E_{r_0}') , \tag{19a}$$

where the sums over l_0 and r_0 are restricted by the bias V. For small voltage the total current is

$$I(V)=\frac{4Ve^2}{\pi\hbar}\frac{\pi^2|\langle H'\rangle|^2\rho_L\rho_R}{|1-|\langle H'\rangle|^2G_LG_R|^2}\ , \tag{19b}$$

where

$$\rho_i=\frac{1}{\pi}\mathrm{Im}G_i,\quad i=(L,R)\ . \tag{19c}$$

$I(V)$ is a maximum when $|\langle H'\rangle|^2$ satisfies $|\langle H'\rangle|^4\times|G_L|^2|G_R|^2=1$ in which case Eq. (19) gives

$$I(V)=\frac{2Ve^2}{\pi\hbar}[(1+\delta_R^2)^{1/2}(1+\delta_L^2)^{1/2}+1-\delta_L\delta_R]^{-1}\ , \tag{20a}$$

where

$$\delta_i=\mathrm{Re}G_i/\mathrm{Im}G_i,\quad i=L,R\ . \tag{20b}$$

$I(V)$ takes its maximum value when the barrier is symmetric in which case $G_L=G_R$ and $I(V)=(2e^2/h)V$ corresponding to a minimum resistance of $h/2e^2$. In addition, it can be shown that the maximum in $I(V)$ occurs when the transmission coefficient is unity. This can only occur if the scanning tunneling microscope makes contact with the sample as opposed to a very small distance from the sample predicted in Ref. 39.

C. Application to resonant tunneling

We continue by applying the formalism to the case of a heterojunction. Let l denote a state localized to the left of the heterojunction, r to the right, and a single state I in the well [see Fig. 1(b)]. We require $U_{R,0}$ which is obtained from Eq. (12),

$$\begin{aligned}U_{R,0}(E)=&H'_{R,0}(E)+H'_{R,I}(E)U_{I,0}(E)\xi(E-E'_I)\\&+\sum_{r\neq(R,0)}H'_{R,r}(E)U_{r,0}(E)\xi(E-E'_r)\\&+\sum_{l\neq(R,0)}H'_{R,l}(E)U_{l,0}(E)\xi(E-E'_l)\ .\end{aligned} \tag{21}$$

We are only interested in "nearest-neighbor" coupling; no intra- or two-step jumps [leading to $E'_s=E_s$ in Eq. (10b)]. Therefore we only keep terms that are lowest order in the overlap $\lambda\equiv\langle l|I\rangle\sim\langle I|r\rangle\sim e^{-Kd}$, where $2d$ is the total width of the tunneling barriers and $\hbar^2K^2/2m=U_0-E$, where U_0 is the barrier height. This results in

$$U_{R,0}(E)=H'_{R,0}(E)+H'_{R,I}(E)U_{I,0}(E)\xi(E-E_I)\ , \tag{22}$$

where $U_{I,0}(E)$ is given by Eq. (12) as

$$\begin{aligned}U_{I,0}(E)=&H_{I,0}+\sum_{l\neq0}H'_{I,l}(E)U_{l,0}(E)\xi(E-E_l)\\&+\sum_r H'_{I,r}(E)U_{r,0}(E)\xi(E-E_r)\ .\end{aligned} \tag{23}$$

To lowest order in the overlap λ one finds

$$U_{l,0}(E)=H'_{l,I}(E)U_{I,0}(E)\xi(E-E_I)\ , \tag{24a}$$

$$U_{r,0}(E)=H'_{r,I}(E)U_{I,0}(E)\xi(E-E_I)\ . \tag{24b}$$

Use of Eq. (24) in Eq. (23) yields

$$U_{I,0}(E)=\frac{H'_{I,0}(E)}{1-\xi(E-E_I)\gamma_I(E)}\ , \tag{25a}$$

where

$$\begin{aligned}\gamma_I(E)=&\sum_{l\neq0}|H'_{l,I}(E)|^2\xi(E-E_l)\\&+\sum_r|H'_{r,I}(E)|^2\xi(E-E_r)\ ;\end{aligned} \tag{25b}$$

and finally use of Eq. (25) in Eq. (22) gives

$$U_{R,0}(E)=H'_{R,0}(E)+\frac{H'_{R,I}(E)H'_{I,0}(E)}{E-E_I-\gamma_I(E)}\ . \tag{26}$$

Equation (26) can be compared to Eq. (5); the main difference is the appearance of the self-energy of the intermediate state γ_I, which prevents $U_{R,0}$ from being singular at resonance. Near resonance, the first term in Eq. (26) can be neglected compared to the second, and using Eq. (9) for $b_R(t)$ we get for the transition rate $\omega_{R,0}=\lim_{t\to\infty}(d/dt)|b_R(t)|^2$,

$$\omega_{R,0}\simeq\frac{2\pi}{\hbar}\frac{|H'_{R,I}(E_0)|^2|H'_{I,0}(E_0)|^2}{(E_R-E_I)^2+\gamma^2}\delta(E_0-E_R)\ , \tag{27a}$$

where neglecting energy shifts gives $\gamma_I=-i\gamma$, $\gamma=\gamma_R+\gamma_L$:

$$\gamma_L(E_0)=\pi\sum_{l\neq0}|H'_{I,l}(E_0)|^2\delta(E_0-E_l)\ , \tag{27b}$$

$$\gamma_R(E_0)=\pi\sum_r|H'_{I,r}(E_0)|^2\delta(E_0-E_r)\ . \tag{27c}$$

Near resonance, Eq. (27a) becomes

$$\begin{aligned}\omega_{R,0}\simeq&\frac{2\pi}{\hbar}\frac{\pi}{\gamma}|H'_{R,I}(E_0)|^2|H'_{I,0}(E_0)|^2\\&\times\delta(E_0-E_R)\delta(E_R-E_I)\ ,\end{aligned} \tag{28}$$

and the total probability current j is

$$j\equiv\sum_{0,R}\omega_{R,0}=\frac{j_R(E_I)j_L(E_I)}{j_R(E_I)+j_L(E_I)}\ , \tag{29a}$$

where

$$j_L(E_I)=\frac{2\pi}{\hbar}\sum_l|H'_{I,l}(E_I)|^2\delta(E_I-E_l) \tag{29b}$$

is the probability current for tunneling from the left-hand side of a junction to the intermediate state I. Similarly,

$$j_R(E_I)=\frac{2\pi}{\hbar}\sum_r|H'_{I,r}(E_I)|^2\delta(E_I-E_r)\ . \tag{29c}$$

From Eq. (29a) it follows that $1/j=1/j_L+1/j_R$ for a double-barrier model. This result was obtained earlier by Payne[2] using a single-barrier model plus current conservation.

Equation (27a) can be evaluated for the case of a symmetric barrier shown in Fig. 1(b). The transmission coefficient can, of course, be evaluated directly by match-

ing wave functions in this simple example and is given by[2]

$$|T(k)|^2=\frac{16K^4k^4e^{-4Kd}}{(K^2+k^2)^2}\left\{\left[(K^2+k^2)\left[\frac{a}{2}+\frac{1}{K}\right]\Delta k+2Kk\left[\frac{K^2-k^2}{K^2+k^2}\right]e^{-2Kd}\right]^2+\left[\frac{4K^2k^2}{K^2+k^2}e^{-2Kd}\right]^2\right\}^{-1}, \tag{30a}$$

where the energy of the incident plane wave is $E=\hbar^2k^2/2m$, $K=[(2m/\hbar^2)(U_0-E)]^{1/2}$ is the wave number in the barrier where U_0 is the barrier height, $\Delta k=k-k_I$ is the distance from resonance where the resonant energy is $E_I=\hbar^2k_I^2/2m$ and is determined by

$$(K^2-k_I^2)\sin(k_Ia)+2k_IK\cos(k_Ia)=0 \tag{30b}$$

where a is the width of the well. Use of the relationship $E-E_I\simeq(\hbar^2/m)k_I\Delta k$ in Eq. (30a) gives

$$|T(k)|^2=\frac{A(E)}{[E-E_I-\Sigma_R(E)]^2+\Sigma_I^2}, \tag{31a}$$

where

$$\Sigma(E)=-\frac{\hbar^2k_I}{m}\frac{1}{\left[\frac{a}{2}+\frac{1}{K}\right]}\frac{2Kk}{(K-ik)^2}e^{-2Kd} \tag{31b}$$

and

$$A(E)=4|\Sigma(E)|^2\frac{K^2k^2}{(K^2+k^2)^2}. \tag{31c}$$

In Eq. (31) terms of higher order than e^{-2Kd} have been neglected where d is the width of each barrier.

Applying the transfer Hamiltonian formalism described above to the symmetric barrier, one finds that γ_I given by Eq. (25b) is equal to $\Sigma(E)$ in Eq. (31b) and that the resonant tunnel current calculated from Eq. (27a) is the same as that calculated by use of the transmission coefficient, Eq. (31a). The total current can easily be put in the form of Eq. (29a). Thus the transfer Hamiltonian method gives an identical result to that obtained by the usual quantum-mechanical calculation for the case of resonance tunneling to order e^{-2Kd}.

D. Application to photoinduced resonant tunneling

In the case where the heterojunction is exposed to an external electromagnetic field the coupling is of the form $2W(\mathbf{x},\mathbf{p})\cos(\omega t)$ and it is treated by generalizing Eq. (8) to

$$i\hbar\dot{b}_s=\sum_n H'_{s,n}(E_n)b_ne^{i(E_s-E_n)t/\hbar}+2\sum_n W_{s,n}b_ne^{i(E_s-E_n)t/\hbar}\cos(\omega t)-i\hbar\sum_{n\neq s}\langle s|n\rangle\dot{b}_ne^{i(E_s-E_n)t/\hbar}+i\hbar\delta_{s,0}\delta(t), \tag{32a}$$

where

$$W_{s,n}\equiv\langle s|W|n\rangle, \tag{32b}$$

and the last term on the right-hand side of Eq. (32a) ensures that an electron is present in the state 0 at time $t=0$.

Following the same steps as in Sec. II C (see the Appendix) the transition probability for going from an initial state 0 on the left to a final state R on the right can be written as a golden rule:

$$\omega_{R,0}(E_R)=\frac{2\pi}{\hbar}\{|f_{R,0}(E_R)|^2\delta(E_R-E_0)+[|g_{R,0}(E_R)|^2\delta(E_R-E_0+\hbar\omega)+(\omega\rightarrow-\omega)]\}, \tag{33a}$$

with

$$f_{R,0}(E)=f^{(0)}_{R,0}(E)+\left[\sum_{l,l'}f^{(0)}_{R,l}(E)W_{l,l'}W_{l',0}\xi(E-E_l)\xi(E-\omega-E_{l'})+\sum_{r,r'}W_{R,r'}W_{r',r}f^{(0)}_{r,0}(E)\xi(E-E_r)\xi(E-\omega-E_{r'})+(\omega\rightarrow-\omega)\right], \tag{33b}$$

where the terms within the large parentheses in Eq. (33b) represent a renormalization of the first term and are caused by photon absorption and emission:

$$f^{(0)}_{R,0}(E)=\frac{H'_{R,I}(E)H'_{I,0}(E)}{E-E_I-\gamma_I}, \tag{33c}$$

where $H'(E)$ is defined in Eq. (10c) and

$$g_{R,0}(E)=\sum_r W_{R,r}f^{(0)}_{r,0}(E+\hbar\omega)\xi(E+\hbar\omega-E_r)+\sum_l f^{(0)}_{R,l}(E)W_{l,0}\xi(E-E_l). \tag{33d}$$

Equation (33) is represented in a graphic form in Fig. 2. The first term $f^{(0)}_{R,0}$ is the tunneling term arrived at earlier. $f_{R,0}$ includes a renormalization of $f^{(0)}_{R,0}$ due to photon emission and reabsorption. $g_{R,0}$ corresponds to photon-assisted tunneling in which a photon is either emitted or absorbed prior to or subsequent to tunneling.

In deriving Eq. (33) we have allowed for absorption only in the electrodes. This is based on the following observation. In semiconductors with a typical doping, the energy scale for the electron gas created by the doping is in the 10–100-meV range. This corresponds to very long

penetration depths, much larger than the tiny extension of the well region. It is therefore possible to neglect the part of the absorption which has only to do with the well. This picture is in contrast to Ref. 35, which stressed the importance of the coupling of the well with the incoming light. In our picture absorption in the bulk electrodes outside of the well creates electron-hole pairs which a bias sweeps through the system and collects in the form of an I-V characteristic. The only influence of the well is to give an energy-selective constriction to electron transport via the resonant level. We thus conclude that any intrinsic high-frequency cutoff of such a system is also related to the optical properties of the electrodes, which in reality often means the optical properties of *bulk* n-type doped GaAs.

In the following section Eq. (33) is used to evaluate the current through an irradiated heterojunction. It will be seen that $f_{R,0}(E)$ in Eq. (33b) represents a renormalization of $f_{R,0}^{(0)}(E)$, the first term on the right-hand side of Eq. (33b).

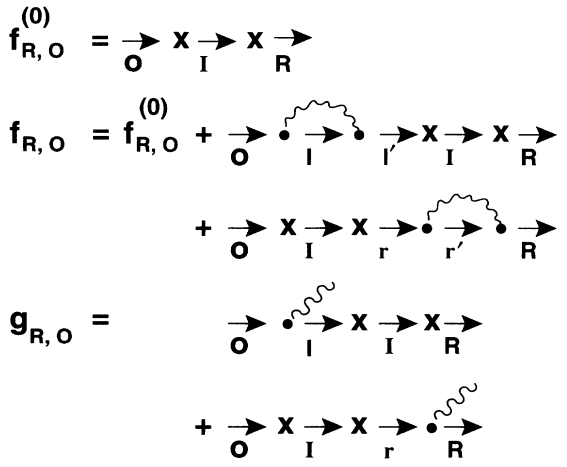

FIG. 2. The processes described in Eqs. (33b)–(33d) are shown diagrammatically. $f_{R,0}^{(0)}$ corresponds to an electron on the left-hand side of the junction in the state 0 tunneling to the resonant level I and then to the state R on the right. $f_{R,0}$ consists of $f_{R,0}^{(0)}$ plus a term that describes emission and reabsorption of a photon followed by tunneling as well as a term where emission and reabsorption occur after tunneling. The terms in $f_{R,0}$ in Eq. (33b) indicated by $(\omega\rightarrow-\omega)$ involve absorption followed by reemission before or after tunneling. The diagrams shown for $g_{R,0}$ correspond to tunneling followed by emission of a photon and photon emission followed by tunneling. Terms with $(\omega\rightarrow-\omega)$ involve photon absorption rather than emission.

III. EXTENDED TRANSFER HAMILTONIAN METHOD APPLIED TO THE SOLLNER EXPERIMENT

A. Experimental parameters

Before using the full transfer Hamiltonian method to describe the irradiated junction we shall review the specific energies characterizing the Sollner experiment.[7]

Sollner *et al.*[7] demonstrated that a biased quantum well in the presence of radiation (2.5 THz) shows a current response which correlates well with a simple small-signal circuit theory analysis. The quantum well used has the structure shown schematically in Fig. 1(b). To assign the various energies, conclusions from the analysis of Goldman, Tsui, and Cunningham[29] have been used. The energy of the intermediate state E_I has been calculated from a well with infinitely thick walls and the Fermi energy E_F of the electrodes is determined from the doping density. The results are $E_I=78$ meV and $E_F=53$ meV using the parameters of the Sollner construction and an effective mass of $0.067m_e$ for the GaAs material. From a perturbation analysis of the level width γ we estimate it to be approximately 1 meV. However, the effective width γ_{eff}, including other broadening mechanisms such as rough barriers, is around 10–20 meV. This is obtained by comparing the Sollner experiment with a theoretical calculation[32] in which γ is estimated from the dc I-V characteristic. γ is thus of the same magnitude as the frequency used in the experiment, 10 meV, which in turn is about a factor of 5 smaller than E_F. For later use, the bulk plasma frequency ω_p of the electron gas in the electrodes is 39 meV based on a doping density of $10^{24}/\text{m}^3$, an effective mass as above, and a background dielectric function of 12.7. However, both E_F and ω_p are very sensitive to the doping level which is usually only given to one significant figure. It should, furthermore, be noticed that for a symmetric barrier we need to apply a voltage of $2\times(78-53)$ meV$=50$ meV to bring the intermediate state into resonance with the left electrode. At this bias the right electrode Fermi energy is almost below the bottom of the left well, hence it is not necessary to worry about the availability of final states in the tunneling process.

B. Theory of photoassisted tunneling

In device physics it is appropriate to calculate the current j_0 $(=I_0/A$, A being the cross-sectional area) which flows through the junction. For the sake of simplicity the junction is assumed to be spatially symmetric. Since j_0 is obtained by an energy integration over $\omega_{R,0}$,

$$j_0=2e\frac{1}{A}\sum_{0,R}\omega_{R,0}\ , \tag{34a}$$

the first part of Eq. (33a) gives the traditional result for the dc I-V characteristic of the junction,[40,41] which we reproduce in order to show the physical ingredients that enter our description. Using the first term in Eq. (33a) for $\omega_{R,0}$ with $f_{R,0}$ replaced by $f_{R,0}^{(0)}$ gives

$$j_0=\frac{4\pi e}{\hbar}\int d^2k_{\parallel}\frac{1}{(2\pi)^2}\int dp\frac{L}{\pi}\int dq\frac{L}{\pi}\left|\frac{H'_{R,I}(E_q)H'_{I,0}(E_q)}{E_q-E_I-\gamma_I}\right|^2\delta(E_q-E_p)\ , \tag{34b}$$

where L is the quantization length of the electrodes and where the momentum parallel to the junction interface, $k_{\parallel}$, is conserved due to translational invariance, and $p(q)$ is the perpendicular momentum in the initial (final) state. $E_I = E_I^0 - eV/2$, where E_I^0 is the position of the intermediate state before a bias is applied. The effect of using the renormalized term $f_{R,0}$ rather than $f_{R,0}^{(0)}$ will be discussed at the end of this section.

Within the transfer Hamiltonian formalism we can relate $H'_{I,0}$ to the transmission coefficient T of the barrier, viz.,[25]

$$|H'_{I,0}|^2 = \left[\frac{\hbar^2}{2m^*}\right]^2 \frac{1}{La} p k_I T(E_p) , \tag{35a}$$

where k_I is defined by

$$E_I^0 = \frac{\hbar^2 k_I^2}{2m^*} \tag{35b}$$

and a is the well width. In what follows we assume the infinite barrier result $k_I = \pi/a$. This leads to a more transparent form for j_0,

$$j_0 = \frac{em^*\gamma^2}{2\pi^2\hbar^3} \int_0^{E_F} dE_p \int_0^{E_F - E_p} dE_{\parallel} \int_{E_F - eV - E_{\parallel}}^{\infty} dE_q \frac{1}{(E_p - E_I)^2 + \gamma^2} \delta(E_q - E_p) , \tag{36a}$$

where γ_I has been approximated by its imaginary part $\gamma = E_I^0 T/\pi$ and γ is assumed to be energy independent over the range of integration. Simplifying Eq. (36a) further yields

$$j_0 \simeq \frac{em^*\gamma^2}{2\pi^2\hbar^3} \int_0^{E_F} dE_p (E_F - E_p) \frac{1}{(E_p - E_I)^2 + \gamma^2} . \tag{36b}$$

The integration in Eq. (36b) can be performed exactly leading to a well-known result (see e.g., Refs. 20, 32, and 41), which we do not reproduce since its detailed form will not be used in our discussion.

Thus the transfer Hamiltonian formalism gives the main features of the static I-V characteristics of a quantum-well tunneling barrier with resonant transmission. Equation (36b) gives the resonant contribution to the current. Incorporating a term $H'_{R,0}$ in Eq. (33c) would produce the traditional Ohmic background current through the junction.

Before calculating the second part of Eq. (33a) we first study the case of photon absorption in one of the electrodes. The energy absorbed per unit time by electrons which reach the tunneling barrier without scattering in a distance l is given by

$$\Delta = \tfrac{1}{2} \mathrm{Re} \int d^3r \, \mathbf{j} \cdot \mathbf{E}^* e^{-z/l} . \tag{37}$$

With $\mathbf{j} = \sigma \mathbf{E}$, σ being the conductivity and assuming $\mathbf{E}$ has the form $\mathbf{E}_0 t e^{-z/\delta}$ where δ is the penetration depth,

$$\Delta = \frac{AE_0^2\omega}{2\Gamma} \epsilon_0 \epsilon_2(\omega) |t|^2 \tag{38a}$$

and

$$\Gamma = \frac{2}{\delta} + \frac{1}{l} , \tag{38b}$$

where A is the cross-sectional area of the sample and ϵ_2 is the imaginary part of the dielectric function. Calculating Δ with the transfer Hamiltonian method involves transitions from a state $(k_{\parallel}, p)$ to a state $(k_{\parallel}, q)$ where $k_{\parallel}$ is the momentum parallel to the barrier; thus

$$\Delta = \hbar\omega \sum_{0,R} \omega_{R,0} = 4\pi A \omega \int d^2k_{\parallel} \frac{1}{(2\pi)^2} \int dp \frac{L}{\pi} \int dq \frac{L}{\pi} |W_{q,p}|^2 \delta(E_q - E_p - \hbar\omega) , \tag{39}$$

where $W_{q,p}$ is the electron-photon matrix element and describes *intraelectrode* absorption.

In order to (i) simulate *intraband* absorption taking place in the doped GaAs electrode, and (ii) to ensure that $\Delta \to 0$ when $\Gamma \to \infty$ as in Eq. (38), define W_0 such that

$$\int dq |L W_{q,p}|^2 \delta(E_q - E_p - \hbar\omega) \equiv \frac{W_0(\omega)}{\Gamma} , \tag{40}$$

and with

$$2 \int d^2k_{\parallel} \frac{1}{(2\pi)^2} \int dp \frac{1}{2\pi} = n ,$$

the electron density in the electrode, Eq. (39), yields

$$\Delta = \frac{2\omega A W_0 n}{\Gamma} , \tag{41a}$$

and therefore the choice

$$W_0 = \frac{\epsilon_2 \epsilon_0 E_0^2 |t|^2}{4n} \tag{41b}$$

reproduces Eq. (38). Because W_0 is proportional to ϵ_2 it is small at frequencies greater than the plasma frequency. This characteristic cutoff frequency only depends on the

doping density of the electron gas.[42] We therefore anticipate an upper cutoff in the response of an irradiated double barrier around 40 meV (or 10 THz) based on a doping of $10^{24}/m^3$ due to the electromagnetic response properties of the *electrodes*. This result is consistent with the experimental findings in Ref. 7 where a THz response from an irradiated double barrier is observed.

Consider the first term in Eq. (33d) which describes the two tunneling events and absorption/emission in the right electrode,

$$g_1=\left[\sum_r W_{R,r}H'_{r,I}(E_0)\xi(E_0-E_r)\right]\frac{H'_{I,0}(E_0)}{E_0-E_I-\gamma_I} . \quad (42)$$

The term in large parentheses contains all transitions between the intermediate state and the right-hand side, a photon event, and a tunneling event. Evaluating the expression in large parentheses in Eq. (42) gives

$$\sum_r W_{R,r}H'_{r,I}(E_0)\xi(E_0-E_r)=-\frac{2im^*L}{p\hbar^2}W_{qp}H'_{p,I}(E_0) , \quad (43)$$

where p (q) denotes the initial (final) -state momentum perpendicular to the junction. Inserting this into Eq. (34a) for the current we can write

$$j_1=\frac{16\pi e}{\hbar}\int d^2k_\parallel\frac{1}{(2\pi)^2}\int dp\frac{L}{\pi}\int dq\frac{L}{\pi}\left[\frac{\hbar^2}{2m^*}\right]^4\frac{1}{L^2a^2}\left[\frac{m^*}{p\hbar^2}\right]^2|LW_{q,p}|^2\frac{p^2k_I^2T^2(E_p)}{(E_p-E_I)^2+\gamma^2}\delta(E_q-E_p+\omega)+(\omega\rightarrow-\omega) , \quad (44a)$$

where we have included the other frequency part in Eq. (33a) and used Eq. (35) to express H' in terms of the transmission factor T. Carrying out the energy integrations in Eq. (44a) and making use of Eq. (40) for the electromagnetic coupling yields

$$j_1=\frac{em^*\gamma^2}{2\pi^2\hbar^3}\int_0^{E_F}dE_p(E_F-E_p)\frac{1}{(E_p-E_I)^2+\gamma^2}\times\sqrt{E_I^0/E_p}\left[\frac{n_0(\omega)}{n}\right]+(\omega\rightarrow-\omega) , \quad (44b)$$

where E_I^0 is the energy of the resonant state when $V=0$ and where we have introduced a dimensionless ratio for the photoexcitation probability through $n_0(\omega)$:

$$n_0(\omega)=\frac{4nW_0(\omega)}{\hbar\Gamma v_I} , \quad (45)$$

where $v_I=\hbar k_I/m^*$. n_0 represents the number of electrons that are photoexcited and then are available for tunneling.

An inspection of Eq. (44b) shows that if $eV\ll|E_I^0|$ the factor $\sqrt{E_I^0/E_p}$ is close to unity and the current can be expressed in terms of the dc I-V characteristic j_0 in Eq. (36b) as

$$j_1(V,\omega)=2j_0(V)\frac{n_0(\omega)}{n} , \quad (46)$$

where the factor of 2 is due to equal contributions from ω and $-\omega$. This result can be understood in a simple way; $j_0(V)$ electrons flow through the junction as in the dc case and they are photoexcited with probability $n_0(\omega)/n$ when they reach the right-hand side. Because $n_0(\omega)\propto\epsilon_2(\omega)$, there will be no photocurrent measured if there is no absorption/emission in the right electrode.

The next contribution in Eq. (33d) which corresponds to an absorption/emission event in the left electrode and two tunneling events gives a contribution j_2 to the total current

$$j_2=\frac{16\pi e}{\hbar}\int d^2k_\parallel\frac{1}{(2\pi)^2}\int dp\frac{L}{\pi}\int dq\frac{L}{\pi}\left[\frac{\hbar^2}{2m^*}\right]^4\frac{1}{L^2a^2}\left[\frac{m^*}{q\hbar^2}\right]^2\times|LW_{q,p}|^2\frac{q^2k_I^2T^2(Eq)}{(E_q-E_I)^2+\gamma^2}\delta(E_q-E_p+\omega)+(\omega\rightarrow-\omega) , \quad (47a)$$

which we rewrite as

$$j_2=\frac{em^*\gamma^2}{2\pi^2\hbar^3}\int_0^{E_F}dE_p(E_F-E_p)\frac{1}{[E_p-(E_I-\hbar\omega)]^2+\gamma^2}\times\sqrt{E_I^0/E_p}\frac{n_0(\omega)}{n}+(\omega\rightarrow-\omega) . \quad (47b)$$

Again, since the factor under the square root is almost unity when $eV\ll E_I^0$, j_2 can be written in terms of j_0 to obtain

$$j_2(V,\omega)\simeq[j_0(V+2\omega)+j_0(V-2\omega)]\frac{n_0(\omega)}{n} . \quad (48)$$

The factors of 2 in Eq. (48) arise because the resonant level is displaced by an amount $V/2$ in a symmetric barrier.

Combining the contributions to the current given by Eqs. (46) and (48) yields

$$j(V,\omega)=\frac{n_0(\omega)}{n}[2j_0(V)+j_0(V+2\omega)+j_0(V-2\omega)] . \tag{49}$$

The result for $j(V,\omega)$ given in Eq. (49) is easily understood. The first term represents tunneling from left to right followed by either photoemission or photoabsorption. $j_0(V)$ describes the tunneling process and $n_0(\omega)/n$ is the probability of emission or absorption. The second term corresponds to emission of a photon with probability $n_0(\omega)/n$ followed by tunneling given by the term $j_0(V+2\omega)$. For a symmetric barrier a potential of strength V lowers the resonant level at energy E_I^0 by $V/2$. In our model an energy loss ω prior to tunneling has the same effect as applying an additional bias of strength 2ω so the tunneling current is proportional to $j_0(V+2\omega)$. Similarly, photon absorption prior to tunneling is equivalent to decreasing the bias V by an amount 2ω and this gives the third term $j_0(V-2\omega)$.

In addition to the photoinduced current $j(V,\omega)$ there is the dc current $j_0(V)$ derived from $f_{R,0}^{(0)}$. The influence of the electromagnetic field changes the magnitude of $j_0(V)$. This is apparent by the appearance of $f_{R,0}$ rather than $f_{R,0}^{(0)}$ in Eq. (33a). Using the full contribution $f_{R,0}$ results in a current $j_0(V)(1-4n_0/n)$ as shown in the Appendix and leads to a total current through the irradiated and biased junction

$$j_T(V,\omega)=j_0(V)+P[j_0(V+2\omega) + j_0(V-2\omega)-2j_0(V)] , \tag{50a}$$

which is obtained by including the contribution given by Eq. (49). An equation of the same form as (50a) has been used by Wingreen[32] to fit the data of Sollner *et al.*[7] The quantity $P\equiv n_0(\omega)/n$ is the ratio of photoexcited current to "normal" current. P is evaluated by use of

$$\tfrac{1}{2}\epsilon_0 E_0^2=\hbar\omega n_p , \tag{50b}$$

where n_p is the number of photons per unit volume and $|t|^2$ is the transmission factor for light penetration of the junction electrodes. Use of Eqs. (41b), (45), and (50b) yields

$$P=4\pi\frac{cn_p}{v_F n}\frac{\epsilon_2|t|^2}{\Gamma\lambda} , \tag{50c}$$

where λ is the wavelength of the light.

Equation (50c) for P can be understood as follows. P can be estimated from the ratio of photoassisted current to the normal current

$$P\sim\frac{\dot{N}_e l}{n v_F} , \tag{51a}$$

where $\dot{N}_e$ is the rate at which electrons are photoexcited per unit volume, l is the electron mean free path, and v_F is the Fermi velocity. $\dot{N}_e$ is given by

$$\dot{N}_e=\frac{\sigma_1 E^2}{2\hbar\omega} , \tag{51b}$$

where E is the field in the junction and σ_1 is the real part of the conductivity. E^2 is related to the number of photons incident on the junction per unit volume by

$$\tfrac{1}{2}\epsilon_0 E^2=\hbar\omega n_p|t|^2 , \tag{51c}$$

where $|t|^2$ is the transmission factor for light in the electrodes, $t=2/(1+\sqrt{\epsilon})$ where ϵ is the dielectric function of the electrodes. Use of Eqs. (51b) and (51c) in Eq. (51a) yields Eq. (50c) for the case $l\ll\delta$, which is the case in Ref. 7.

In the case that j_0 varies slowly in the energy ranges from $V-2\omega$ to $V+2\omega$, Eq. (50a) gives

$$j_T(V,\omega)\simeq j_0(V)+4P\omega^2\frac{\partial^2}{\partial V^2}j_0(V) , \tag{52}$$

in agreement with the small-signal analysis as discussed below.

C. Small-signal analysis

Let $I_0(V)$ be the (known) dc I-V characteristic. If we modulate V with a small potential $v\cos\omega t$ representing an external microwave field we can expand $I_0(V+v\cos\omega t)$ around V. The time-averaged change in the current response is $\Delta I\equiv I-I_0=\frac{1}{4}v^2 I_0''$, where I_0'' is the second derivative of I_0 at the working point. This means that the extra current is proportional to the second derivative of the static I-V characteristic. Sollner's experimental results are similar in form to the second derivative and demonstrate a response at high frequency.

Assuming that the applied field can be represented as a varying potential difference between the left- and right-hand sides of the junction, Tien and Gordon[37] have given a general treatment for an irradiated Josephson junction which is also valid for other diodelike systems in the presence of an external potential. They demonstrated that in the presence of a microwave field $v\cos\omega t$, one can solve exactly for the wave function and density of states in the presence of the radiation which gives the following form for the current in terms of the original dc-characteristic $I_0(V)$:

$$I(V,\omega)=\sum_{n=-\infty}^{n=\infty}J_n^2(\alpha)I_0\left[V+\frac{n\hbar\omega}{e}\right] , \tag{53}$$

where the coupling strength is $\alpha\equiv ev/\hbar\omega$. Expanding Eq. (53) where I_0 is slowly varying over the scale of ω yields the small-signal result discussed above. Thus those equations are relevant to the experimental results reported by Sollner *et al.*

In the limit $\alpha\ll 1$, Eq. (53) takes the same form as Eq. (50a) with P replaced by $\alpha^2/4$. The ratio of these factors can be written as

$$r\equiv\tfrac{1}{4}\alpha^2/P\simeq(d/d_0)^2 , \tag{54a}$$

where d is a fictitious barrier thickness defined by

$$d=\frac{v_0}{E} \tag{54b}$$

and

$$d_0=2\sqrt{\hbar/m^*\omega}\ , \tag{54c}$$

where m^* is the effective mass of the electrodes and the relations $\epsilon_2\approx(1/\omega\tau)(\omega_p/\omega)^2$ and $l=v_F\tau$ have been used.

IV. SUMMARY

An extended transfer Hamiltonian method capable of treating resonant tunneling junctions has been presented. This was required because the expression for the resonant tunneling current as derived by the usual transfer Hamiltonian method is divergent. Our results for the current in the presence of a photon field are expressed in terms of the current in the absence of photons and are similar in form to those of Ref. 37, which used a spatially constant potential to describe the coupling of the electrons to the photons. We give an expression for the coupling strength in Eq. (50c). The ratio of the coupling strengths for the two approaches is given by Eq. (54). In the experiment of Sollner *et al.*[7] the ratio is of order unity, but it will normally be quite different depending on the experimental conditions.

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APPENDIX

In this appendix, Eq. (33) is derived from Eq. (32). Use of Eq. (9) in Eq. (32) yields

$$\begin{aligned}(E-E_s)G_{s,0}(E)&=\sum_{n\neq s}H'_{s,n}(E)G_{n,0}(E)\\&\quad+\sum_n W_{s,n}[G_{n,0}(E+\omega)+G_{n,0}(E-\omega)]+\delta_{s,0}\ ,\end{aligned} \tag{A1}$$

where $G_{n,0}(E)$ is defined by Eq. (9a). Defining $U_{n,0}(E)$ by Eq. (11a) then leads to

$$\left[U_{s,0}(E)-H'_{s,0}(E)-\sum_{n\neq s,0}H'_{s,n}(E)U_{n,0}(E)\xi(E-E_n)\right]G_{0,0}(E)=\left[\left[W_{s,0}+\sum_{n\neq 0}W_{s,n}U_{n,0}(E+\omega)\xi(E+\omega-E_n)\right]G_{0,0}(E+\omega)+(\omega\rightarrow-\omega)\right]\ . \tag{A2}$$

Equation (A2) shows that $G_{0,0}(E)$ is coupled to $G_{0,0}(E\pm\omega)$ through W. For $s=0$, Eq. (A1) yields

$$\left[E-E_0-\sum_{n\neq 0}H'_{0,n}(E)U_{n,0}(E)\xi(E-E_n)\right]G_{0,0}(E)=1+\left[\left[W_{0,0}+\sum_{n\neq 0}W_{0,n}U_{n,0}(E+\omega)\xi(E+\omega-E_n)\right]G_{0,0}(E+\omega)+(\omega\rightarrow-\omega)\right]\ , \tag{A3}$$

which replaces Eq. (13a)

Define

$$t_{s,0}(E+\omega)=W_{s,0}+\sum_{n\neq 0}W_{s,n}U_{n,0}(E+\omega)\times\xi(E+\omega-E_n)\ . \tag{A4}$$

Equation (A2) yields

$$U_{R,0}(E)=H'_{R,0}(E)+H'_{R,I}(E)U_{I,0}(E)\xi(E-E_I)+\left[t_{R,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right]\ , \tag{A5}$$

where

$$G_\pm=G_{0,0}(E\pm\omega)\ . \tag{A6}$$

In what follows, translational invariance parallel to the junction dictates that all quantities are a function of the conserved wave number $k_\parallel$, which we do not write out explicitly.

Again using Eq. (A2),

$$\begin{aligned}U_{I,0}(E)&=H'_{I,0}(E)+\sum_{l\neq 0}H'_{I,l}(E)U_{l,0}(E)\xi(E-E_l)\\&\quad+\sum_r H'_{I,r}(E)U_{r,0}(E)\xi(E-E_r)\\&\quad+\left[t_{I,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right]\ ,\end{aligned} \tag{A7}$$

as well as

$$U_{r,0}(E)=H'_{r,I}(E)U_{I,0}(E)\xi(E-E_I)+\left[t_{r,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right] \tag{A8}$$

and

$$U_{l,0}(E)=H'_{l,I}(E)U_{I,0}(E)\xi(E-E_I)+\left[t_{l,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right], \tag{A9}$$

where $H'_{l,0}$ and $H'_{r,0}$ have been neglected because we include only nearest-neighbor hopping. Use of Eq. (25b) for the intermediate-state self-energy in Eqs. (A7)–(A9) gives

$$U_{I,0}(E)=\left[H'_{I,0}(E)+\sum_{l\neq0}H'_{I,l}(E)\left[t_{l,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right]\xi(E-E_l)+\sum_r H'_{I,r}(E)\left[t_{r,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right]\xi(E-E_r)+\left[t_{I,0}(E+\omega)\frac{G_+}{G_{0,0}}+(\omega\rightarrow-\omega)\right]\right]\frac{1}{1-\xi(E-E_I)\gamma_I} \tag{A10}$$

and therefore to lowest order in W

$$U_{R,0}(E)G_{0,0}(E)=f_{R,0}(E)G_{0,0}(E)+\left[\frac{H'_{I,0}(E)}{E+\hbar\omega-E_I-\gamma_I}\sum_{r'}W_{R,r'}H_{r',I}(E+\omega)\xi(E+\hbar\omega-E_{r'})G_{0,0}(E+\hbar\omega)+(\omega\rightarrow-\omega)\right]+\left[\frac{H'_{R,I}(E)}{E-E_I-\gamma_I}\sum_{l\neq0}H'_{I,l}(E)W_{l,0}\xi(E-E_l)G_{0,0}(E+\hbar\omega)+(\omega\rightarrow-\omega)\right], \tag{A11a}$$

where

$$f_{R,0}(E)=\frac{H'_{R,I}(E)H'_{I,0}(E)}{E-E_I-\gamma_I}+\frac{H'_{R,I}(E)}{E-E_I-\gamma_I}\left[\sum_{l,l'}H'_{I,l}(E)W_{l,l'}W_{l',0}\xi(E-E_l)\xi(E+\omega-E_{l'})+(\omega\rightarrow-\omega)\right]+\left[\sum_{r,r'}W_{R,r'}W_{r',r}H'_{r,I}(E)\xi(E-E_r)\xi(E+\omega-E_{r'})+(\omega\rightarrow-\omega)\right]\frac{H'_{I,0}(E)}{E-E_I-\gamma_I}. \tag{A11b}$$

The first term in $f_{R,0}$ is independent of W, while the second and third terms are $O(W^2)$. The transition probability $\omega_{R,0}$ involves the square of $f_{R,0}$ and to lowest order this produces terms $O(W^0)$ and $O(W^2)$. The term that is $O(W^2)$ must be kept because the second and third terms in Eq. (A11a) give terms that are $O(W^2)$. Use of Eq. (A11) in Eq. (14) yields the final result, Eq. (33). There is an additional term in Eq. (A11b) that has been neglected. This term involves photon absorption (or emission) on the left followed by photon emission (or absorption) on the right of the barrier. When calculating the total current this term is canceled by the cross term obtained from $|g_{R,0}(E)|^2$ in Eq. (33d).

Finally, it will be shown how the photon corrections to $f^{(0)}_{R,0}$ change the ordinary dc current. Using Eq. (43) and $E_R\simeq E_I\simeq E_0$ the correction terms in Eq. (A11b) can be written as

$$f_{R,0}(E)-f^{(0)}_{R,0}(E)=\frac{H'_{R,I}(E_R)H'_{I,0}(E_R)}{E_R-E_I-\gamma_I}\sum_{l'}|W_{l',0}|^2\xi(E_0+\omega-E_{l'})\left[\frac{-2im^*L}{p\hbar^2}\right]+\frac{H'_{I,0}(E_R)H'_{R,I}(E_R)}{E_R-E_I-\gamma_I}\sum_{r'}|W_{R,r'}|^2\xi(E_R+\omega-E_{r'})\left[\frac{-2im^*L}{p\hbar^2}\right]+(\omega\rightarrow-\omega), \tag{A12}$$

where $E_R=\hbar^2p^2/2m^*$. Use of Eqs. (40) and (45) in Eq. (A12) yields

$$|f_{R,0}(E)|^2=|f^{(0)}_{R,0}(E)|^2\left[1-\frac{4n_0}{n}\right] \tag{A13}$$

to lowest order in W, which demonstrates that j_0 in Eq. (34b) should be multiplied by a factor $1-4n_0/n$. These considerations yield Eq. (50).

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